Single Pure – Surds

Given a choice between $\sqrt{8}$ and $2\sqrt{2}$ patrons should prefer $2\sqrt{2}$. Also square roots should never be left on the denominator of a fraction in a final answer.

1. Write the following in the form $a\sqrt{b}$ where b is as small as it can be:

   (a) $\sqrt{8}$. $\boxed{2\sqrt{2}}$
   (b) $\sqrt{27}$. $\boxed{3\sqrt{3}}$
   (c) $\sqrt{12}$. $\boxed{2\sqrt{3}}$
   (d) $\sqrt{45}$. $\boxed{3\sqrt{5}}$
   (e) $\sqrt{450}$. $\boxed{15\sqrt{2}}$
   (f) $\sqrt{2}\sqrt{24}$. $\boxed{4\sqrt{3}}$
   (g) $\sqrt{a^4b^2c}$. $\boxed{a^2b\sqrt{c}}$
   (h) $\sqrt{ab^4c^3d^{100}}$. $\boxed{b^2cd^{50}\sqrt{ac}}$
   (i) $\sqrt[3]{54}$. $\boxed{3\sqrt[3]{2}}$
   (j) $\sqrt{8}+\sqrt{18}$. $\boxed{5\sqrt{2}}$
   (k) $\sqrt{12}+\sqrt{75}-\sqrt{48}$. $\boxed{3\sqrt{3}}$
   (l) $\sqrt{20}+\sqrt{5}-\sqrt{45}$. $\boxed{0}$
   (m) $\sqrt{200}-\sqrt{288}+\sqrt{128}$. $\boxed{6\sqrt{2}}$

2. Write the following in the form $\sqrt{a}$:

   (a) $5\sqrt{2}$. $\boxed{\sqrt{50}}$
   (b) $4\sqrt{3}$. $\boxed{\sqrt{48}}$
   (c) $2\sqrt{5}$. $\boxed{\sqrt{20}}$
   (d) $7\sqrt{6}$. $\boxed{\sqrt{294}}$
   (e) $\dagger^2\sqrt{\bullet}$. $\boxed{\sqrt{\dagger^4\bullet}}$
   (f) $3\sqrt[3]{x+1}$. $\boxed{\sqrt[3]{27x+27}}$
   (g) $a\sqrt[n]{k}$. $\boxed{\sqrt[n]{a^n k}}$
   (h) $\sqrt{12}+3\sqrt{3}$. $\boxed{\sqrt{75}}$
   (i) $\sqrt{8}+\sqrt{2}$. $\boxed{\sqrt{18}}$
   (j) $\sqrt{45}-\sqrt{20}$. $\boxed{\sqrt{5}}$
   (k) $\sqrt{108}+\sqrt{12}-\sqrt{3}$. $\boxed{\sqrt{147}}$
   (l) $\sqrt{44}-\sqrt{99}+\sqrt{176}$. $\boxed{\sqrt{99}}$
   (m) $\sqrt{3a^2}+\sqrt{12b^2}-\sqrt{3a^2+3b^2-6ab}$. $\boxed{\sqrt{27b^2}}$

3. Expand and simplify fully:

   (a) $(2\sqrt{3})^3$. $\boxed{24\sqrt{3}}$
   (b) $(\sqrt{18})^3$. $\boxed{54\sqrt{2}}$
   (c) $(2-\sqrt{3})^2$. $\boxed{7-4\sqrt{3}}$
   (d) $(1-\sqrt{8})^2$. $\boxed{9-4\sqrt{2}}$
   (e) $(4+\sqrt{32})^2$. $\boxed{48+32\sqrt{2}}$
   (f) $(3+\sqrt{20})^3$. $\boxed{207+94\sqrt{5}}$

4. Rationalise the denominator and simplify fully:

   (a) $\dfrac{4}{\sqrt{2}}$. $\boxed{2\sqrt{2}}$
   (b) $\dfrac{1}{\sqrt{3}}$. $\boxed{\frac{\sqrt{3}}{3}}$
   (c) $\dfrac{25}{2\sqrt{5}}$. $\boxed{\frac{5\sqrt{5}}{2}}$
   (d) $\dfrac{21}{6\sqrt{7}}$. $\boxed{\frac{\sqrt{7}}{2}}$
   (e) $\dfrac{ab}{c\sqrt{b}}$. $\boxed{\frac{a\sqrt{b}}{c}}$
   (f) $\left(\dfrac{2}{\sqrt{3}}\right)^3$. $\boxed{\frac{8\sqrt{3}}{9}}$
   (g) $\dfrac{2}{\sqrt{2}}+\dfrac{\sqrt{2}}{3}$. $\boxed{\frac{4\sqrt{2}}{3}}$
   (h) $\dfrac{3}{\sqrt{5}}-7\sqrt{5}$. $\boxed{\frac{-32\sqrt{5}}{5}}$
   (i) $\dfrac{1}{\sqrt{8}}+\dfrac{1}{\sqrt{2}}$. $\boxed{\frac{3\sqrt{2}}{4}}$
   (j) $\sqrt{\dfrac{1}{2}}+\sqrt{\dfrac{1}{4}}+\sqrt{\dfrac{1}{8}}$. $\boxed{\frac{2+3\sqrt{2}}{4}}$
   (k) $\dfrac{1}{\sqrt{2}-1}$. $\boxed{\sqrt{2}+1}$
   (l) $\dfrac{4}{\sqrt{3}+1}$. $\boxed{2(\sqrt{3}-1)}$

(m) $\dfrac{4-\sqrt{3}}{2+\sqrt{3}}$. $11-6\sqrt{3}$

(n) $\dfrac{1-\sqrt{5}}{\sqrt{5}+2}$. $3\sqrt{5}-7$

(o) $\dfrac{\sqrt{2}+3}{1-2\sqrt{2}}$. $-1-\sqrt{2}$

(p) $\dfrac{\sqrt{7}+1}{\sqrt{7}-1}$. $\frac{4+\sqrt{7}}{3}$

(q) $\dfrac{a}{b+c\sqrt{k}}$. $\frac{a(b-c\sqrt{k})}{b^2-c^2k}$

(r) $\dfrac{9}{\sqrt{3}-1}+\dfrac{7}{\sqrt{3}+1}$. $8\sqrt{3}+1$

(s) $\dfrac{\sqrt{2}+3}{\sqrt{2}-1}+\dfrac{1}{\sqrt{2}}$. $\frac{10+9\sqrt{2}}{2}$

(t) $\left(\frac{1}{1-\sqrt{2}}\right)^2+\left(\frac{2}{3+\sqrt{2}}\right)^2$. $\frac{191+74\sqrt{2}}{49}$

(u) $\dfrac{69}{1+\sqrt{2}+\sqrt{3}}$.

5. Solve the following equations and simultaneous equations:

(a) $\sqrt{3}x+1=0$. $x=-\frac{\sqrt{3}}{3}$

(b) $\sqrt{2}x-2=x$. $x=2(1+\sqrt{2})$

(c) $\sqrt{4}x-3=\sqrt{3}x$. $x=6+3\sqrt{3}$

(d) $\sqrt{3}x=4+2x$. $x=-8-4\sqrt{3}$

(e) $\sqrt{5}x+1=x-3$. $x=-1-\sqrt{5}$

(f) $\begin{matrix}\sqrt{5}x+y=1\\3x+2y=2\end{matrix}$. $(x,y)=(0,1)$

(g) $\begin{matrix}\sqrt{2}x-2y=0\\x+\sqrt{2}y=1\end{matrix}$. $(x,y)=(\frac{1}{2},\frac{\sqrt{2}}{4})$

(h) $\begin{matrix}x-\sqrt{5}y=0\\\sqrt{5}x+y=1\end{matrix}$. $(x,y)=(\frac{\sqrt{5}}{6},\frac{1}{6})$

(i) $\begin{matrix}3x+\sqrt{2}y=7\\\sqrt{2}x+5y=-1\end{matrix}$. $(x,y)=(\frac{35+\sqrt{2}}{13},-\frac{3+7\sqrt{2}}{13})$

(j) $\begin{matrix}3x+2y=5\\x-\sqrt{2}y=4\end{matrix}$. $(x,y)=(1+\sqrt{2},\frac{2-3\sqrt{2}}{2})$

(k) $\begin{matrix}\sqrt{2}x+y=4\\x+2y=1\end{matrix}$. $(x,y)=(2\sqrt{2}+1,-\sqrt{2})$

(l) $\begin{matrix}\sqrt{7}x+y=5\\2x+\sqrt{7}y=-4\end{matrix}$. $(x,y)=(\frac{5\sqrt{7}+4}{5},-\frac{4\sqrt{7}+10}{5})$

(m) $\begin{matrix}\sqrt{5}x+\sqrt{5}y=4\\3x-\sqrt{5}y=1\end{matrix}$. $(x,y)=(\frac{5\sqrt{5}-15}{2},\frac{75-47\sqrt{5}}{10})$

(n) $\begin{matrix}\sqrt{3}x+y=2\\x-3y=1\end{matrix}$. $(x,y)=(\frac{21\sqrt{3}-7}{26},\frac{7\sqrt{3}-11}{26})$

(o) $\begin{matrix}\sqrt{3}x-y=2\\4x+3y=1\end{matrix}$. $(x,y)=\left(\frac{21\sqrt{3}-28}{11},\frac{41-28\sqrt{3}}{11}\right)$

(p) $\begin{matrix}\sqrt{a}x-y=1\\x+3y=\sqrt{a}\end{matrix}$. $(x,y)=(\frac{3a+8\sqrt{a}-3}{9a-1},\frac{3a\sqrt{a}-a-3\sqrt{a}+1}{9a-1})$